The universe is considered in this article, arising from two base vectors (1,0), (0,1), arranged in form of a 2x2-matrix. The two matrices obtained by interchanging the vectors are for an identity matrix $id$ and the first Pauli matrix $\sigma_1$ from quantum theory. The group is commutative and used for the conjugation operator $C$ of physics and its square $id$. For the characteristic polynomial $z-1$ with root 1 it is the symmetry on a dihedral $D_1$ which has on its circle one point 0. The symmetry generates a cw clockwise or mpo counter clockwise rotation on $D_1$.

A plane $E$, generated by the two base vectors, has real numbers for its orthogonal $uv$-coordinate axes. For the first coordinate setting the real numbers are scaling $u \cdot id$, $u = x$ or $u = r$ (radius) and imaginary numbers $iv \cdot C$, $v = ct$, $t$ time, $c$ speed of light. The rt-plane is extended to a projective plane either $P^2$ by adding a real line at infinity $S^1$ or by a complex 1-compactification to a Riemannian sphere $S^2$. Both versions are used later on.

Take first $P^2$. Since Einstein proposed successfully for physics Minkowski metric, a correlation of $P^2$ is given by the 2x2-matrix with base $(0,1)$ (first row), $(1,0)$ (second row) which is the second Pauli matrix $\sigma_2$. The correlation associates with a point $(r,ct)$ the line $ar – bct = 0$ and the metrical quadric $r^2 – c^2t^2 = 0$ for $(a,b) = (r,ct)$. These are two lines, intersecting at 0. At infinity they are closed by two points to a lemniscate. If rotated in higher dimensions they generate the Minkowski light cone for special relativity as an affine geometry, disregarding the part at projective infinity. The symmetry is given by the Lorentz transformations of special relativity.

Take now the complex closure $S^2$ of a plane. Quantum measures use Gleason operators $T$. 2-dimensions are extended by the real cross product to 3 dimensions. The $T$'s spin-like orthogonal base triple (frame) of space is chosen for spanning the Gleason frames $GF$ unit sphere $S^2$. For scaled coordinates it has an ellipsoid quadric $f(s) = ax^2+by^2+cz^2$ with positive real scalars $a,b,c$. For a spin $GF$ $a=b=c$ measures spin length. There can be two or three values for $a,b,c$. Since a $GF$ attains its maximum $M$ and minimum $m$ values the possibilities are for a suitable frame $c = M$, $a=m=b$ and $a<b<c$. Three values can occur for measuring mass of particle series. For two values the angle $\Theta$ is determined with $x = \cos \Theta$ as north pole and $y = \sin \Theta = z$ on the equator of $S^2$.

For the extension of the $GF$ scaling number system from reals to complex, quaternionic and octonian coordinates the reduced base matrices $id$ and $\sigma_2$ are used. For complex numbers the $\sigma_2$ matrix multiplies like the imaginary number $i$ with $i^2 = 1$. In the above $GF$ the complex matrix presentation for the linear $x$-coordinate is $id$ and $\sigma_2$ for $y$ and the complex numbers are written in matrix form as $x \cdot id + z \cdot \sigma_2$. A $GF$ can now have complex scalars $a,b,c$ with the homogeneous norming $f(\lambda u) = f(u)$ for $u = (x,y,z)$ and complex $x,y,z,\lambda$ with $|\lambda| = 1$. The complex polar coordinates are constructed and allow with the exponential function $exp(i\phi)$ wave characters like $a \cdot exp(i\omega t)$ for energies. Particle characters are due to real $GF$ which set for instance (heat or) mass scalars at barycenters, sometimes in triples, using a Higgs field for mass and Higgs bosons. The polar $U(1)$ circle is a dihedral $D_0$ with $id$ as symmetry and characteristic polynomial a real constant like 1 or $2\pi$ for the circles radius or length. The WI weak interactions symmetry $SU(2)$ adds to the Pauli matrix $\sigma_2$ the quaternion 2x2-matrices $\sigma_1$, $\sigma_3$ for the space coordinates $x,z$ where $y$ is the $\sigma_2$ coordinate. For the quaternion scalars of the $GF$ the real scalars $x,z$ of the complex numbers are replaced by adding the two other Pauli matrices where the scalars are for $x \cdot id + z \cdot \sigma_2 + iy \cdot \sigma_3 + ict \cdot \sigma_1$ with the first row $(z_1 = x+iy,$
$z_2 = z^+ict)$ and the second row $(-z^+ict_2, x-iy)$. On a latitude circle of the Hopf $S^2$ can rotate an energy charge like the electrical e0. This is a dihedral $D_1$ with the symmetry id, C, the conjugation operator of physics. The characteristic polynomial is $z-1$. $D_1$ sets the possible clockwise mpo $+e0$ or counterclockwise $-e0$ rotations on $D_2$. An angular momentum for orbiting energies is generated. If the energy is observed in wave form, the wave length has to fit to the $D_2$ latitude which is measured by the above setting of the angle $\theta$ and spin length. Only quantized radii for main quantum integer numbers $n = 1, 2, \ldots$ have wave solutions. If the charges energy is increased or decreased, from the angular momentum a scaled linear frequency for photons can be released or absorbed and these field quantums allow then to change the latitude of $D_1$. These rotations for systems can be seen as a flat oriented orbital character about a barycenter. As Kepler’s conic sections the leaning circle $\theta$ for the $S^2$ GF presentation is kept, but the quadrif for $S^3$ is changed by a projective rescaling whose operator shifts infinity such that the Minkowski cone quadratic $x^2 + y^2 - z^2 = 0$ is obtained. The transversal plane $E$ cuts the cone for $\theta = 0$ in a circle in $E$ as an equatorial plane, leaning with $\theta$ the plane against the cone gives the Kepler ellipses, if $E$ is parallel to a cone surface line through its center an escape parabola is the orbit; leaning further until $E$ is parallel to the cone axis gives escape hyperbolas as orbits. The Einstein rescaling of ellipses to rosette orbits is discussed in [1]. The quaternion scaled GF allow CPT Klein/Pauli subgroups $Z2xZ2$ of order 4 id, C, the conjugation operator $\sigma_j$ for $C$, $P$, $T$ operators. A as discussed in [1] the Heisenberg uncertainty couplings on the signed $x, y, z$ axes rays are for differentiating functions, using the differentials for length or radius $dx$, $dr$, for time $dt$ area $dA$ and volume $dV$.

The complex GF’s are 6-dimensional, the quaternionic 12-dimensional as real projective Hilbert spaces. The numbers for coordinate dimensions in matrix presentation are extended from real 1 to complex 2 dimensions, to 4 for quaternions by the Pauli matrices and id which pairs two complex numbers $z$ to $(z_1, z_2)$. Extended for octonians the Cayley Dickson construction repeats this pairing for quaternions $q$ replacing complex numbers to octonians $(q_1, q_2)$. The multiplication table of octonians is different from the strong interactions SU(3) matrices. It fits for seven GF’s like the real spin GF for measuring length. The strong SI interaction extends the weak symmetry SU(2) to SU(3) having the gluons 3x3-Gell-Mann matrices $\lambda_j$, $j = 1, \ldots, 8$, as generators. There is postualted that gravity acts also with these matrices. The SU(3) geometry is a trivial fiber bundle $S^3xS^5$ of unit spheres. The first $\lambda 1, 2, 3$ matrices as rgb-graviton spin-like whirls project the $S^3$ factor down to the weak Hopf geometry $S^1$ in spacetime. $S^5$ is discussed latr on.

The octonion coordinates are written by their indices $0, 1, 2, \ldots 7$ for a coordinated space of 8-dimensions in which not only spacetime 1234 lives, but also the physics forces like WI, SI with their matrix extensions. It is postulated that the octonion 56-energy plane is for frequency $f$ and mass $m$ with the Einstein line $mc^2 = hf$, $h$ the Planck constant. The linear 7-coordinate is rolled to a projective $S^1$ circle for the U(1) symmetry of the electromagnetic interaction and force with photons as field quantums. It introduces the before mentioned exponential function $\exp(i\varphi)$, $\varphi$ a complex measured polar angle for polar coordinates $x+iy = r\exp(i\varphi)$. The octonion $e_0$ coordinate is of use for setting vectors in the octonion vector space at any place with a measuring unit attached for the line and +-orientation generated by the local vector on it. To the SI 123456 space are added six energies:

\[ z_2 = z^+ict \] and the second row \((-z^+ict_2, x-iy)\).

On a latitude circle of the Hopf $S^2$ can rotate an energy charge like the electrical $e_0$. This is a dihedral $D_1$ with the symmetry id, C, the conjugation operator of physics. The characteristic polynomial is $z-1$. $D_1$ sets the possible clockwise mpo $+e0$ or counterclockwise $-e0$ rotations on $D_2$. An angular momentum for orbiting energies is generated. If the energy is observed in wave form, the wave length has to fit to the $D_2$ latitude which is measured by the above setting of the angle $\theta$ and spin length. Only quantized radii for main quantum integer numbers $n = 1, 2, \ldots$ have wave solutions. If the charges energy is increased or decreased, from the angular momentum a scaled linear frequency for photons can be released or absorbed and these field quantums allow then to change the latitude of $D_1$. These rotations for systems can be seen as a flat oriented orbital character about a barycenter. As Kepler’s conic sections the leaning circle $\theta$ for the $S^2$ GF presentation is kept, but the quadrif for $S^3$ is changed by a projective rescaling whose operator shifts infinity such that the Minkowski cone quadratic $x^2 + y^2 - z^2 = 0$ is obtained. The transversal plane $E$ cuts the cone for $\theta = 0$ in a circle in $E$ as an equatorial plane, leaning with $\theta$ the plane against the cone gives the Kepler ellipses, if $E$ is parallel to a cone surface line through its center an escape parabola is the orbit; leaning further until $E$ is parallel to the cone axis gives escape hyperbolas as orbits. The Einstein rescaling of ellipses to rosette orbits is discussed in [1]. The quaternion scaled GF allow CPT Klein/Pauli subgroups $Z2xZ2$ of order 4 id, C, the conjugation operator $\sigma_j$ for $C$, $P$, $T$ operators. A as discussed in [1] the Heisenberg uncertainty couplings on the signed $x, y, z$ axes rays are for differentiating functions, using the differentials for length or radius $dx$, $dr$, for time $dt$ area $dA$ and volume $dV$.

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electrical and mass charge (15) generating potentials a/r as a driving force POT, kinetic and rotational energy (63) associated with speed \( v = dx/dt \) and \( \omega = d\phi/dt \) and heat, magnetic energies (24) where the integrations to perform show that in the magnetic case an area is integrated (or differentiated for induction) and the same for heat, but for a 3-dimensional volume where pressure as force is generated on the volumes surface through heat motions of matter inside. The 6-fold way is due to the triangle symmetry \( D_3 \). It sets in a commutative G-version these six energies and also six color charges, six electrical charges and six masses for the two fermionic series. In the MINT-Wigris Tool bag one tool is the G-compass. G is a 2x2 matrix of order 6 and is a needle of the compass which can be turned only in the direction of the sixth roots of unity. The G-matrix arises from the scaled Einstein's general relativistic Schwarzschild factor with first row vector \((1,-1)\) and second row vector \((0,1)\). The original electrical and mass potentials POT force energy inside a dark matter bifurcates in a decay of the dark matter to the heat/magnetic energies and kinetic/rotational energies. After this Feigenbaum bifurcation the 8 SI gluons bifurcate, then the heat chaos occurs. Physics sets after a dark matter decay the Planck constants as measuring units though the octonian e0 vector: meter/length 1, second 4 time, energy/frequency Joule or inverse second 6, heat Kelvin scaled energy 2, mass 5 (Schwarzschild and Minkowski rescaled later on) in kg, density. Rotational energy 3 measures are derived from them as many other units and the electrical measures distribute also to the octonian coordinates. The seven octonian GF’s are available for energies and U(1), and the SU(3) GF of rgb-gravitons is added to them.

Earlier the \( D_3 \) symmetry was mentioned. Its characteristic polynomial is \( z^3 - 1 \). For the POT force, the electrical potential has spin plus magnetic momentum as an GF. The magnetic field quantum as whirl is replaced for gravity by the rgb-graviton whirl, a superposition of three r,g,b color charge whirls it sets a terahedron geometry with the GF tip as one vertex. The endpoints of the GF vectors have a quark attached. Quarks can be considered as replacing field quantums for the POT field. They carry an electrical and a mass charge. This describes nucleons spanned 3-dimensional by the GF and a quark triangle with symmetry \( D_3 \). The dihedral \( D_2 \) is for quarks with its 2 poles on a circle.

Its characteristic polynomial is \( z^2 - 1 \). The quarks geometry is different as a 1-dimensional lemniscate with two foci for its electrical and mass poles. It is a retract of a flow or field in 2- (or 3-) dimensional for a (solid) brezel of genus 2. The \( D_2 \) version has as symmetry the CPT group with T,C as reflections and P a 180° rotation. As normal subgroup of the tetrahedron symmetry \( S_4 \) of a nucleon it can factor the group to \( D_3 \). The six permutations \( i,j,k = 1,2,3 \), of \( D_3 \) are the cross ratio invariants under the Moebius transformations MT of the \( S^2 \) symmetry. To each ijk its equivalence class has four members, the symmetry ijk, an octonian coordinate 1,2,...,6, an energy as in the WI case above and a color charge, replacing leptonic charges. The SI rotor of \( [1] \) drives integrations with the differentials for radius or length, time, wrea, volume. The gluon interchange between paired quarks take the third quark as tip of a conic color charge whirl, rotating similarly as a magnetic field quantum whirl. The conic whirl rotation with angular frequency is taken as a thrid whirl character for energy beside the particle and wave character. The conic whirl rotates cw with the green color charge tip and mpo with the red color charge.

Figure 1 quark triangle of a nucleon with the rotated momentum vectors

The blue color charge whirl of the rgb-graviton always rotates with the second (not tip) quark. Rotated is their triangle side perpendicular to the triangle plane as diameter of the cone. The motion of blue is such that a harmonic oscillation with three nodes for the quark vertices as a membran oscillation of the triangle surface occurs and that the vertices have 0 momentum attached as fixed points. The cones axes are from one side to a barycenter on the opposite side and are barycentrical coordinates which set at their intersection the nucleon (rescaled) mass.
Beside the unit sphere $S^1$ the SI geometry has $S^5$ as fiber bundle space with $S^1$ as fiber and projection $CP^2$, a complex inner spacetime for atomic kernels. For their nucleon parts are observed inside $CP^2$ three quarks, always of neutral color charge red $r$, green $g$, blue $b$. Repeated is: As whirls they are in superposition for the rgb-gravitons which have them like energy vectors added at the 3-dimensional spin-like three generating base vectors. The $D_3$ symmetry acts as an SI rotor for integrations and gravity is not independent of this (see the Tool bag). Nucleons triangles are taken as base of a tetrahedron with the $S_4$ symmetry of order 24, it factors through the Klein group CPT of order 4 to $D_3$. The $D_3$ matrices are obtained as cross ratios invariants under Moebius transformations. The MT symmetry belongs to a Riemannian sphere $S^2$ as boundary of $CP^2$.

The geometries of particles, whirls and waves are too long to be described here. They belong to the MINT-Wigris project, found in the internet under [3]. Spin, magnetic flow quants, rgb-gravitons have whirls like most quasiparticles. For generating waves in the above octonian setting it is observed that the exponential wave function $a \cdot \exp(i\omega t)$, a real, $t$ time, $\omega$ angular frequency, for a harmonic oscillation uses the earlier mentioned complex $\exp(i\phi)$ construction. The cosine and sinus projections are also used. For the particle character the Schwarzschild radius setting and spacial volumes inside a sphere $S^2$ ar geometric locations, for the whirl character two kinds of geometry present dark whirl and dark energy. The exponential function is taken on a transversal cut through a U(1) rolled elliptic cylinder which is closed at infinitly $\propto$ to a pointed torus Inside dark energy can have higher speeds than light by inversing universes matter/energy speeds. If $\propto$ is replaced by a Minkowski light cone with its two boundaries identified with the cylinders boundary a whirls pointed Horn torus is generated. The toroidal Hopf $S^3$ geometry allows Horn torus with a singular point where diametrical opposite transversal circles touch. This is taken as location of dark matter inside its Schwarzschild radius sphere as boundary. Quarks are retracted from 3-dimensions to the central 1-dimensional lemniscate and joined at the Horn torus singularity of their figures singular point. In the Horn torus decay the singularity is removed and the quarks get their own singular point. This is why matter in atoms consists of quarks and not antiquarks. The antiparticles are then in other decays also less stable in the universe than particles, but occur occasionally also in stable systems like a positron as electrical charge of a proton.

References

